PERGAMON

Vision Research 41 (2001) 53–55

Vision Research

www.elsevier.com/locate/visres

Letter to the Editor

Assumptions concerning orthogonality in threshold-scaled versus cone-contrast colour spaces

A recent article by D'Zmura and Knoblauch (1998) argued from experiments using sectored noise that chromatic detection is subserved by multiple post-receptoral mechanisms, and not solely by mechanisms tuned to specific cone-opponent 'red–green' and 'blue–yellow' directions as has been proposed previously. An important feature of their analysis is their use of angles within a threshold-scaled cardinal colour space to determine the response of a mechanism to a given stimulus. In this letter, we point out that the angle between the colour direction of two stimuli depends on the space in which the stimuli are represented. Moreover, the choice of each space embodies specific assumptions about chromatic detection. Specifically, the use of threshold-scaled cardinal space in the sectored noise study assumes that the visual response arises from the linear combination of postreceptoral outputs. In this letter, we use a more standard assumption of chromatic detection, that of linear summation at the cone level, to demonstrate that their data are consistent with earlier findings that chromatic detection is served by two fundamental mechanisms. This letter, as a whole, addresses an important issue regarding the appropriate use of colour spaces in quantitative analysis and its dependence on underlying assumptions concerning chromatic detection mechanisms.

D'Zmura and Knoblauch's sectored-noise study follows from previous experiments that use to investigate the mechanisms subserving chromatic detection (e.g. Gegenfurtner & Kiper, 1992; Giulianini & Eskew, 1997; Sankeralli & Mullen, 1997). In all these experiments, the detection threshold of a fixed test is measured in the presence of a variable chromatic noise mask. In the sectored-noise approach of D'Zmura and Knoblauch, the noise mask consists of two orthogonal vector components: a fixed component N_T coincident in direction with the test vector, and a variable component N_P perpendicular to this direction in the chosen colour space (Fig. 1). They demonstrate that the variable noise component N_P has no masking effect on the test, and therefore correctly conclude that the underlying detection mechanism is orthogonal to the chromatic direction of this noise component. Adopting a threshold-scaled cardinal space, they argue that this orthogonal mechanism direction always coincides with the direction of the test, and not with fixed red–green and blue–yellow axes as observed in previous studies (Sankeralli & Mullen, 1997; Eskew, McLellan, & Giulianini, 1999).

Firstly, it is clear, that establishment of an orthogonal direction in a given colour space is dependent on the choice of colour space itself. This is illustrated in Fig. 2. In Fig. 2(a), the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are set in orthogonal directions in the (x, y) plane. In Fig. 2(b), the x-axis is rescaled by a factor of 1/5. The vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are now no longer orthogonal; indeed, they are now almost parallel. To explain this algebraically, suppose that we have a 3D stimulus vector $\boldsymbol{s}$ and a derived 3D mechanism vector $\boldsymbol{m}$ that is orthogonal to $\boldsymbol{s}$ in a given space. By the orthogonality relationship, $\boldsymbol{s}^{\mathrm{T}}\boldsymbol{m} = 0$, where $\boldsymbol{s}^{\mathrm{T}}$ denotes the transpose of $\boldsymbol{s}$. If we transpose to a new space through a transformation represented by the 3×3 matrix $\mathbf{A}$, then the new stimulus and mechanism vectors are $\mathbf{A}\boldsymbol{s}$ and $\mathbf{A}\boldsymbol{m}$, respectively. These two vectors in our new space are orthogonal if and only if the cross product $(\mathbf{A}\boldsymbol{s})^{\mathrm{T}}(\mathbf{A}\boldsymbol{m})$ is equal to 0. Evaluating this cross product:

$$(\mathbf{A}\boldsymbol{s})^{\mathrm{T}}(\mathbf{A}\boldsymbol{m}) = \boldsymbol{s}^{\mathrm{T}}(\mathbf{A}^{\mathrm{T}}\mathbf{A})\boldsymbol{m} \quad (1)$$

Given that $\boldsymbol{s}^{\mathrm{T}}\boldsymbol{m} = 0$, Eq. (1) evaluates to zero only if $\mathbf{A}^{\mathrm{T}}\mathbf{A} = \mathbf{I}$, the identity matrix, which in turn requires that $\mathbf{A} = (\mathbf{A}^{\mathrm{T}})^{-1}$, i.e. that the transformation matrix $\mathbf{A}$ is identical to its inverse transform. This requirement is the algebraic definition of an orthonormal transformation: a transformation under which the angles between any two vectors are preserved.

The principle of orthogonality used by the sectored noise study is founded on an established model of the postreceptoral mechanisms (see Eskew, McLellan, & Giulianini, 1999). In this model, it is assumed that the response R of each postreceptoral mechanism is the weighted sum of the cone responses (L, M, S) to a given stimulus: i.e.

$$R = w_L L + w_M M + w_S S) \quad (2)$$

This expression leads to the orthogonality property of *nulling*: given a stimulus vector $\boldsymbol{s}$ with associated cone

PII: S0042-6989(00)00224-8

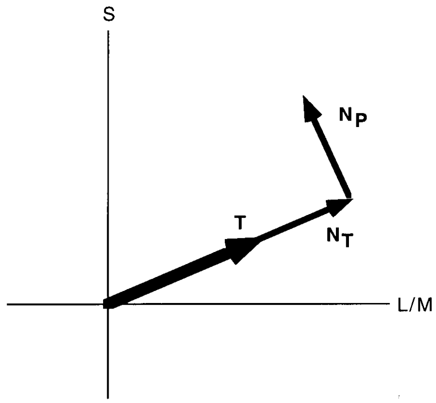

Fig. 1. Vector representation of the sectored-noise stimulus. T represents the test direction; N_T and N_P, the respective components of the noise mask parallel (fixed) and perpendicular (variable) to the test direction in the chosen colour space.

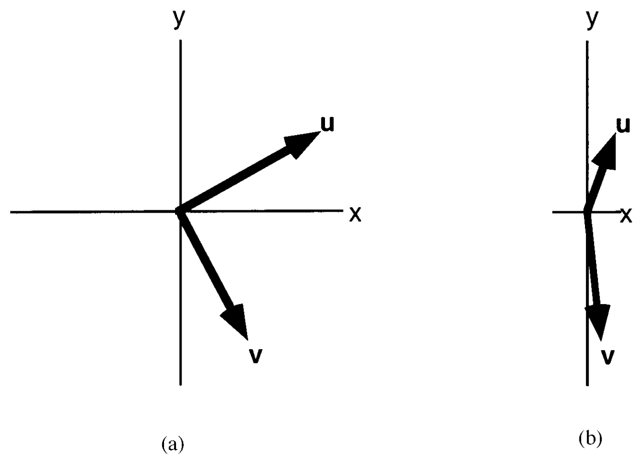

Fig. 2. Effect of axis scaling on orthogonality. The vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal in (a), but are almost parallel in (b) where the x-axis has been resealed by a factor of 1/5.

responses (L, M, S) and a mechanism vector $\boldsymbol{m}$ whose elements are the fixed cone-input weightings (w_L, w_M, w_S) of the mechanism, there is no response of the mechanism to the stimulus ($R=0$) when the vectors $\boldsymbol{s}$ and $\boldsymbol{m}$ are orthogonal. The stimulus in this case is called a null stimulus for that mechanism. It is critical

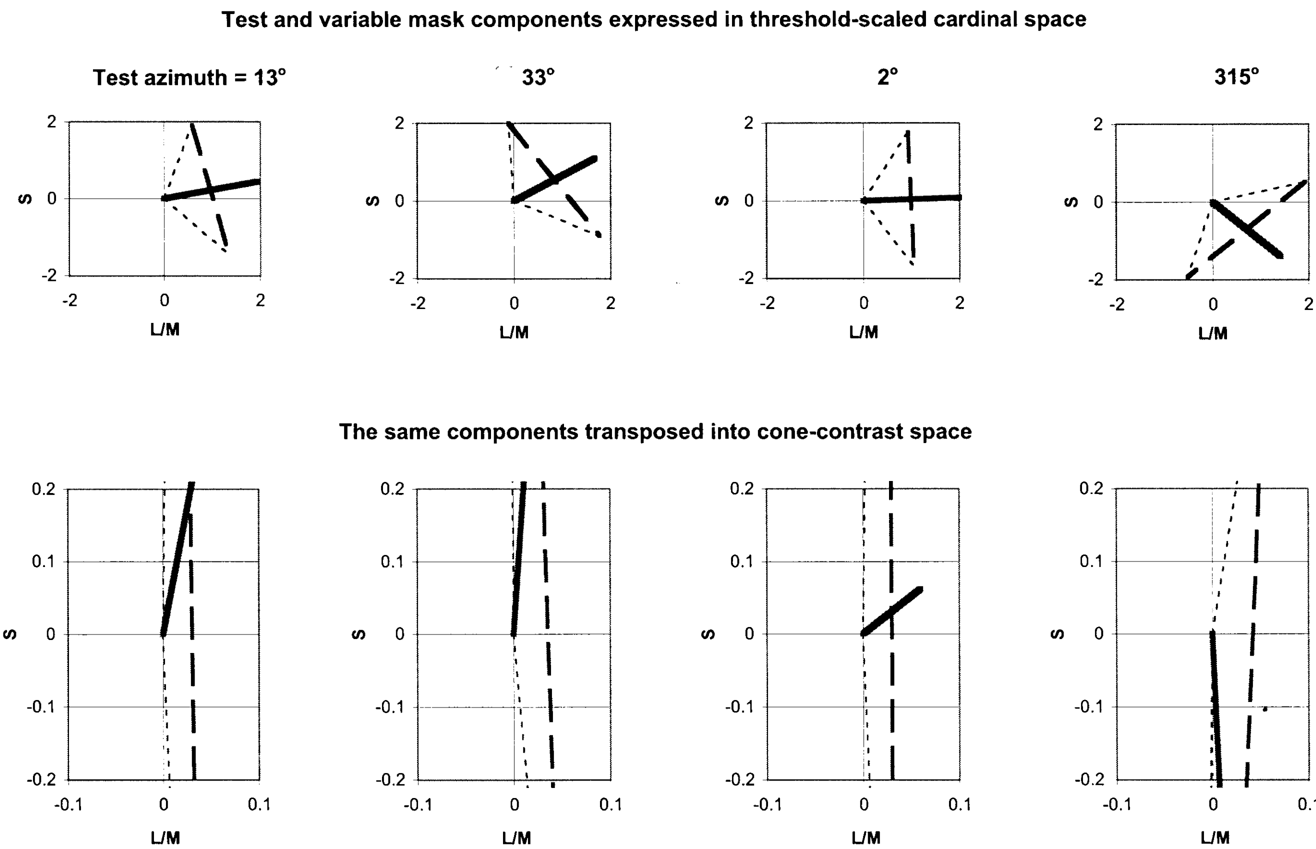

Fig. 3. Test and mask stimuli as represented by vectors in threshold-scaled cardinal space (top row) and following transformation into a cone-contrast space (bottom row). In each panel, the thick, solid line represents the test direction, and the heavy, dashed line, the direction of the variable component of the mask. The thin, dotted lines represent the limits of the 60° half-width noise sector. Each column of panels represents one of the four test conditions used by D'Zmura and Knoblauch (1998). The azimuth of the test in cardinal space is indicated at the top of each column. The figure shows that the orthogonality relationship between the test and mask in threshold-scaled cardinal space is not valid following transformation into cone-contrast space, and that, in cone-contrast space, the variable component of the mask is always near (within 2° of) the S-cone axis.

to note that this orthogonality property was applied originally to cone contrast space (see Cole, Hine, & McIlhagga, 1993), and, from the preceding discussion, is valid only for this space or any orthonormal transformation of this space.

The threshold-scaled cardinal space used in the sectored-noise study is not an orthonormal transformation of cone contrast space. While it is true that the threshold-scaled cardinal space used in the sectored-noise study preserves the directions of the cardinal axes in cone contrast space, Fig. 2 clearly shows that, to obtain an orthonormal transformation, the units of axis length must also be preserved under the transformation. This is not the case in the colour space used in the sectored-noise study, where the L/M axis unit is 0.030 ($=0.021 \times \sqrt{2}$) cone contrast units, whereas that for the S-axis is 0.89 cone contrast units. As a result of this re-scaling of the cardinal axes, the orthogonality principle cannot be applied in this threshold-scaled cardinal space.

Fig. 3 shows the four test directions (azimuth 13, 33, 2 and 315°) used in the sectored-noise study. In each case, we show the test direction (thick, solid line) and the chosen mask direction (heavy, dashed line). The thin, dotted lines show the limits of the sectored-noise stimulus corresponding to a 60° half-width. In the top row, the test and noise vectors are plotted in the colour space chosen in that study. In the bottom row, they are replotted in a colour space having the same axial directions as in the top row, but defined in cone-contrast units. The figure shows that, as represented in cone-contrast units, the variable noise component $\boldsymbol{N}_{\mathrm{P}}$ (heavy, dashed lines) lies, not orthogonal to the test direction, but parallel (within 2°) to the S-axis. Following the logic of the sectored-noise study, the ineffectiveness of $\boldsymbol{N}_{\mathrm{P}}$ in masking the test demonstrates that the test is being detected, not by a mechanism tuned to the test direction, but by a broadband mechanism tuned in a direction orthogonal to $\boldsymbol{N}_{\mathrm{P}}$, i.e. tuned near the L/M axis. This agrees with previous results showing that such test stimuli are detected by a ‘red–green’ mechanism having opponent L- and M-cone weightings (Stromeyer, Cole, & Kronauer, 1985; Cole et al., 1993; Sankeralli & Mullen, 1997; Giulianini & Eskew, 1997).

Sectored-noise analysis provides a useful method for investigating the postreceptoral mechanisms responsible for chromatic detection. In analysing the data produced by this method, however, two factors must be recognised: firstly, the interpretation of these data depends critically on the colour-space representation used; secondly, this colour representation must accurately reflect the underlying principles of detection. If it is assumed that each mechanism response arises from the combination of normalised cone responses, the appropriate colour space must represent these cone responses directly. From the data obtained in the sectored-noise study, a cone-contrast representation will reveal, not mechanisms tuned to the various test directions, but, in all cases, a single detection mechanism having opponent L- and M-cone inputs and little S-cone input. More generally, this letter highlights the importance of an appropriate selection of representation in all multi-dimensional analyses relying on angular measures: a consideration that is not restricted to experiments involving noise masking or chromatic detection.

References

Cole, G. R., Hine, T. J., & McIlhagga, W. (1993). Detection in L-, M-, and S-cone contrast space. *J. Opt. Soc. Am. A*, *10*, 38–51.

D’Zmura, M., & Knoblauch, K. (1998). Spectral bandwidths for the detection of color. *Vis. Res.*, *38*, 3117–3128.

Eskew, R. T., Jr, McLellan, J. S., & Giulianini, F. (1999). Chromatic detection and discrimination. In K. R. Gegenfurtner, & L. T. Sharpe, *Colour vision: from molecular genetics to perception*. Cambridge, UK: Cambridge University Press.

Gegenfurtner, K. R., & Kiper, D. C. (1992). Contrast detection in luminance and chromatic noise. *J. Opt. Soc. Am. A*, *9*, 1880–1888.

Giulianini, F., & Eskew, R. T., Jr (1997). Chromatic masking in the (ΔL/L, ΔM/M) plane of cone-contrast space reveals only two detection mechanisms. *Vis. Res.*, *38*, 3913–3926.

Sankeralli, M. J., & Mullen, K. T. (1997). Postreceptoral chromatic detection mechanisms revealed by noise masking in three-dimensional cone contrast space. *J. Opt. Soc. Am. A*, *14*, 2633–2646.

Stromeyer, C. F., III, Cole, G. R., & Kronauer, R. E. (1985). Second-site adaptation in the red–green chromatic pathways. *Vis. Res.*, *25*, 219–237.

June 26, 2000

Marcel J. Sankeralli,
Kathy T. Mullen
Department of Ophthalmology
McGill Vision Research
McGill University
687 Pine Avenue West
H4-14 Montreal
Québec
Canada H3A 1A1
E-mail: kmullen@violet.vision.mcgill.ca